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Analog Methods

by

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I INTRODUCTION

I appreciate this special opportunity of reviewing analog methods for an audience of this character. It is especially an honor since my company is one of many competitive manufacturers of electronic analog equipment, to say nothing of other companies manufacturing process control equipment. Naturally, I am bound by your trust to put aside my overwhelming preference for Philbrick gear and to be entirely impartial. My assignment here is to discuss quantitatively, in fairly specific terms, how to perform typical analog computor operations such as addition, subtraction, division, multiplication, etc. with pneumatic as well as electronic equipment.

From the most elegant publications all the way to conservations on the subway, much is said of computors, implying that they are huge, awesome beasts too complicated for any except a special cult of engineers to understand. Sometimes it is implied that they are glamorous machines which will be doing most of our thinking for us. Perhaps today we can bring these machines a little closer to home. Computing machines of the analog type are actually simple at the engineering level, in spite of their glamorous front panels. Mr. G. A. Philbrick has observed that they are a friendly, natural phenomena like ourselves though not nearly as complicated, and that probably they have a good deal more to fear from us than we from them.

First and foremost, analog computing is really another expression for model building. Every analog computor is truly a model, both by definition and in every other practical sense. In application to process controls particularly, we think of every operation as being a model of some part of a physical process. The construction of scale models is certainly a part of the analog art. Generally speaking, such models perform in the same physical media as do the original objects. Usually, the scale model is substantially smaller than its prototype, the handier size being one of the principal reasons for going to the trouble of making it. But, in any case, space dimensions undergo a transformation usually in a definite ratio in passing from the original to the model.

One of the most wonderful features of the world of nature around us is the direct analogy from one system to another. For example, you have all learned in dimensional analysis courses that inductance is the direct electrical analogy to mass; that is, they both have identical dimensional notation. Similarly, capacitance is the direct electrical analogy to spring constant and, likewise, resistance is the direct electrical analogy to loss, such as friction-producing elements or viscosity.

On this basis, it becomes possible to build a true model in one physical medium of an original physical process, or machine, etc., in another physical medium. For example, a system comprising a spring, a mass, and a dash pot can be directly, exactly, modeled in an electrical medium, using a capacitance, an inductance, and a resistance.

For the present, let it be considered that almost any process which is quantitatively understood can be modeled on an analog computor. This is made possible by nature's wonderful system of structural parallelism in which a physical element in one medium may be represented by a corresponding physical element in another.

Heart and core of the present day analog art is the "operational amplifier." When the operational amplifier is *well* understood, much of the analog art becomes almost self-evident. This is particularly true of simulator design and of process control. It is generally the amplifier which "puts the numbers" on both system performance capability and on reliabilty.

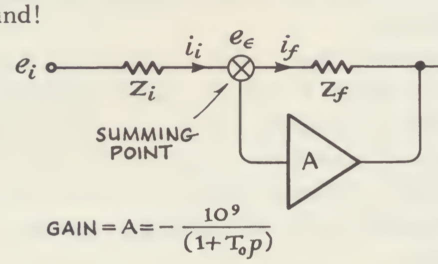
In order to give you something of value this morning by which you can assess analog capability along with some idea of cost, I will emphasize the operational amplifier. Analog circuit techniques amount to little more than connecting these together with a few passive elements, usually in rather standard configurations.

II THE OPERATIONAL AMPLIFIER

The expression "analog methods" or "analog techniques" is generally considered to include only a special class of feedback circuit devices and their accessories. Although active devices could certainly be used in analog modeling, almost all practical analogs used today are characterized by passive networks — I repeat "passive," entirely "passive" networks — balanced around some node, or null. Be it a hydraulic or pneumatic or electronic analog, some sort of activator (or amplifier) is used to force the null to exist at its appointed position in the network.

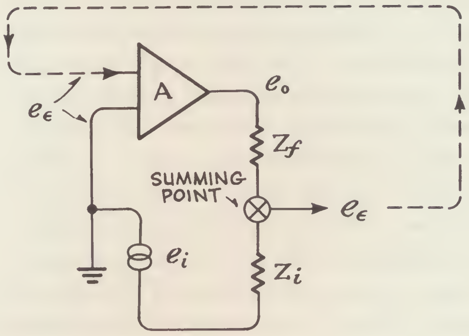
Fig. 1 illustrates just such an analog. It is a passive network activated by an amplifier. The fact that electrical notation is used in it is irrelevant. The system depicted could just as well be hydraulic or pneumatic. Exactly the same equations can be developed for all three; exactly the same principles are utilized. However, it is somewhat easier to explain the concept to engineers of all backgrounds when using the electrical example. To make the example dramatic, but nonetheless real, let us consider that the activator, or amplifier, is one of the new high-reliability types, made for military applications, whose design center dc gain is approximately 10^9; i. e., one *billion* volts per volt. If there were no further considerations involved in such an amplifier, considering the signal requirements for such a gain, one could correctly

infer that when the output swings between +100 volts and −100 volts, the summing point (or input) will have an excursion of less than *one-tenth of a microvolt* each side of ground!



A. CONVENTIONAL PRESENTATION IN COMPUTOR LITERATURE

the way from zero to +90. Let us be yet more dramatic and say that at very low frequencies the sign of the error signal even reverses! Under such extreme conditions, the summing point dc error could now approach one



B. CONVENTIONAL PRESENTATION IN SERVO-SYSTEM LITERATURE

FIGURE 1. BASIC CIRCUIT TYPICAL FOR OPERATIONAL AMPLIFIERS

The best way to visualize any servo-system is to think backwards. Since e_o is the quantity desired, assume that it is, in fact, at the value desired; say, +100 volts. We can now safely infer that the error voltage at the summing point must, at least theoretically, be −0.1 microvolt referred to ground. If this were not the case, the output could not be where we assumed it; namely, at +100 volts.

Keeping our example on the dramatic side but still practical, let us say that the full scale signal to be amplified (input voltage) e_i is −10 millivolts when the full scale output voltage is to be +100 volts. First of all, we can see now that we are demanding a closed loop gain of −10,000 from e_i to e_o. Secondly, we can see that the input current flowing through Z_i must be

(1) $$i_i = \frac{e_i - e_\epsilon}{Z_i} \cong i_f = \frac{e_\epsilon - e_o}{Z_f}$$

Even though e_i is only 100 millivolts, it is still 100,000 times larger than e_ϵ, so we can safely neglect the term e_ϵ and say that the input current is simply e_i/Z_i. If no current flows into the operational amplifier (a pretty good assumption), it is clear that to a very close approximation, indeed, the gain must be determined by the ratio of the two passive elements, Z_f and Z_i:

(2) $$\frac{e_o}{e_i} \cong -\frac{Z_f}{Z_i}$$

Suppose next that the amplifier exhibits atrocious distortion, say over 90% nonlinear distortion, and that 10 times as much error signal is required to drive the output from +90 to +100 volts as is required to drive it all whole microvolt. But the input e_i, is 10 millivolts. So the input current is still in error less than 0.01%, and likewise the output voltage due to this cause.

The first question usually raised is, "How is it possible to close a loop around an amplifier with a gain of one billion volts per volt?" Referring to the standard teachings on servomechanisms, any amplifying system which has a smooth 6 db/octave* rolloff will be stable if the loop is closed around it. Not only will it be stable, but it will be more than critically damped since the phase shift will be just 90° in theory. Note in Fig. 2 that the crossover frequency; i.e., the frequency at which the gain of the amplifier drops to 0 db, or unity gain, is about 10^7 radians per second (about 1.6 megacycles) and the rolloff is a smooth 6db/octave slope starting at 0.1 radians per second. The dc gain is clearly 160 db (100 million volts per volt) (dashed response curve in Fig. 2). Suppose now that we modify this attenuation diagram to show 180 db gain at 0.1 radian/sec. This can be accomplished by letting it fall off at 12 db/octave from 0.1 radians/sec to 1 radian/sec, then at this point go back to 6 db/octave slope. As shown in Fig. 2, the gain will be 10^9 or a billion volts per volt from dc to 0.1 radians/sec. Fig. 2 represents an approximation of the actual frequency response overall of the high-reliability amplifier just discussed.

To illustrate closed-loop frequency response as a function of loop gain, let us consider the amplifier con-

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* db = decibels. octave = change by a factor of 2. An amplifier with a smooth 6 db/octave rolloff is equivalent to an ideal, uniform response amplifier driving a simple R-C low pass filter (see Fig. 2).

nected as a gain of 1,000 amplifier. This is simply another way of saying that the ratio of Z_f to Z_i is 1000 at all frequencies. Except for noise and power limitations, it is irrelevant what the value and the nature of these impedances may be. From Fig. 1B, we see that the computing network acts as an attenuator between the output and the summing point. The division fed back, e_ϵ / e_o, is traditionally labeled β, and so the attenuation factor, by which e_o is reduced to e_ϵ, is the familiar old term, $1/\beta$. This will be covered in greater detail in a moment, but for the present we have arbitrarily assigned the term $1/\beta$ the following value: $\frac{1}{\beta} = 1 + \frac{Z_f}{Z_i}$

Pursuing the example cited, $Z_f/Z_i = 1000$ and so $1/\beta = 1001$, or 60 db. If the open loop gain of the amplifier is 180 db, then after attenuating it some 60 db by the computing network, 120 db is the net gain left around the closed loop.

Turning now to Fig. 3, we can see how it is possible to add three voltages, $e_i + e_a + e_b$. But, in truth, it is the three currents they produce which are added instead. These currents are summed in the summing point junction and, again, can leave only through Z_f. Once again we say that Z_f transduces the sum of these three currents into a voltage, e_o.

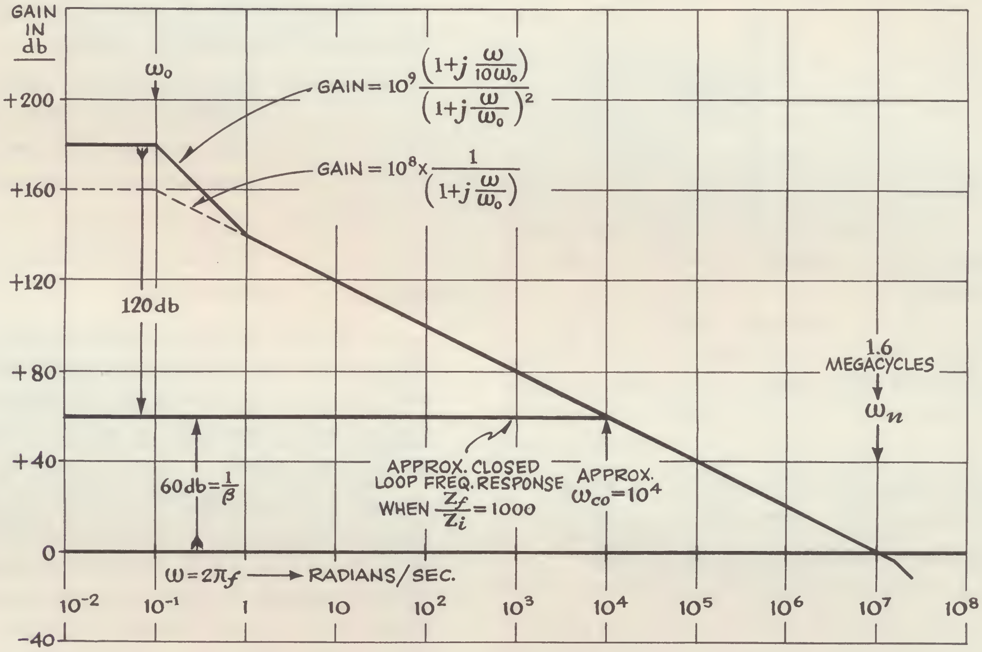


FIGURE 2. SIMPLIFIED FREQUENCY RESPONSE OF HIGH PERFORMANCE OPERATIONAL AMPLIFIER

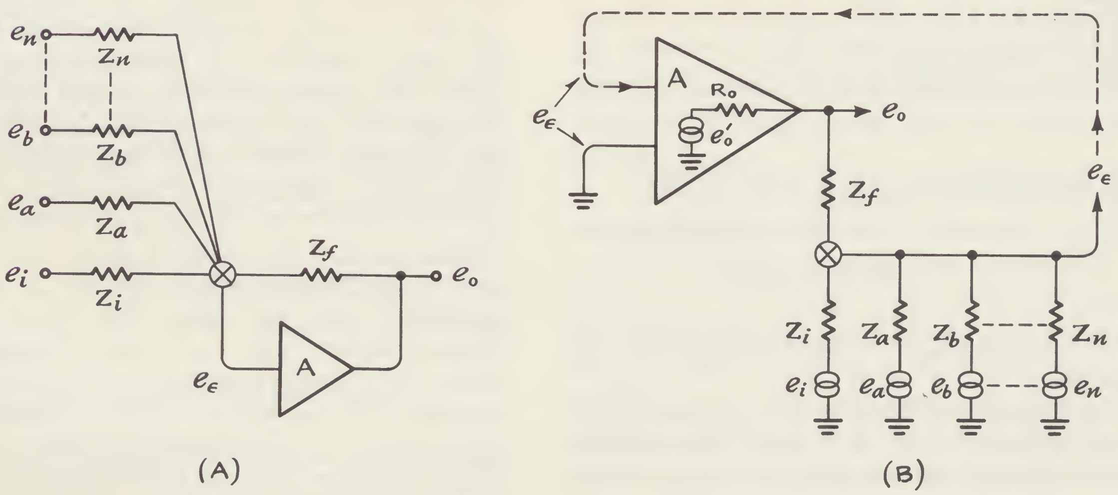


FIGURE 3. TWO PRESENTATIONS OF A GENERALIZED CIRCUIT

High gain is the one thing which makes it possible to isolate the effects of the three different inputs from each other. In effect, it makes the summing point a zero resistance virtual zero volt terminal. Equation (3) represents the generalization of equation (1) and its solution in terms of amplification and impedances. It is generally more convenient to consider only one input at a time. This is indicated in equation (4) which is the more useful form in circuit applications. Since "A" is generally in quadrature throughout the working range, complex numbers or operational notation should be used.

$$\text{(3)}\quad e_o = -\underbrace{\left[e_i\frac{Z_f}{Z_i} + e_a\frac{Z_f}{Z_a} + e_b\frac{Z_f}{Z_b} \text{----} + e_n\frac{Z_f}{Z_n}\right]}_{\text{OUTPUT FOR IDEAL AMPLIFIER}}\underbrace{\left[\frac{1}{1-\frac{1}{A}\left(1+\frac{Z_f}{Z_i}+\frac{Z_f}{Z_a}+\frac{Z_f}{Z_b}\text{----}+\frac{Z_f}{Z_n}\right)}\right]}_{\text{ERROR FACTOR BY WHICH OUTPUT DEVIATES FROM IDEAL}}$$

Dc "error factors" of one part per million (1 ppm) are routinely obtained. But at 100 kilocycles, an error factor which is reliably under 0.2% in magnitude and 4 degrees in phase error is considered good, even for the USA-4 type amplifier.

The factor $1/\beta$, by which the output voltage, e_o, is attenuated to produce the "servo system" error voltage, e_ϵ, can be derived from Fig. 3B in its most general form:

$$\text{(4)}\quad \frac{1}{\beta} = \frac{\Delta e_o}{\Delta e_\epsilon} = \underbrace{\left(1+\frac{Z_f}{Z_i}+\frac{Z_f}{Z_a}+\frac{Z_f}{Z_b}\text{----}+\frac{Z_f}{Z_n}\right)}_{\text{ATTENUATION FACTOR}}$$

When calculating or estimating deviations from perfection, the all-important item to keep in mind is the actual value of the term βA, especially at the highest signal frequency of interest. It is not going too far at all to say that 70% of all disappointments and failures encountered in customer's applications have occurred simply because the value of the term βA has not been considered quantitatively. Note that the "error factor" in equation (3) can now be re-expressed in simpler form:

$$\text{(5)}\quad \text{ERROR FACTOR} = \frac{1}{1-\frac{1}{\beta A}} = 1 - \underbrace{\frac{1}{1-\beta A}}_{\text{ACTUAL ERROR}}$$

$$\text{"} \qquad \text{"} \quad = approx\left(1+\frac{1}{\beta A}\right)$$

Bear in mind that β is a negative value if the feedback is negative.

If the input circuits in Fig. 3 are all passive except for e (i.e., e_a, e_b, . . . e_n are all zero), then equation (3) can be reduced to the much simpler and convenient form:

$$\text{(6)}\quad \frac{e_o}{e_i} = -\frac{Z_f}{Z_i}\underbrace{\left(\frac{1}{1-\frac{1}{\beta A}}\right)}_{\text{ERROR FACTOR}}$$

The only error source cited so far is that error required to activate the amplifier's output. In all hydraulic, pneumatic, electronic, etc., amplifiers there are other forms of error besides just the error signal required to activate the output. These other forms are almost always lumped together and referred to as "noise," meaning spurious, unwanted, uninvited signals.

Noise apparent in operational amplifiers falls into two separate and distinct classes:

Class A: Noise in output due to an apparent NOISE VOLTAGE in series between the summing point and the actual input terminal of an idealized amplifier.

Class B: Noise in output due to an apparent NOISE CURRENT, fed into the summing point.

It should be emphasized that in using these concepts, distortion, even in staggeringly large doses, is not and can never be treated as a spurious input or "noise." Furthermore, distortion is rarely a consideration at all, as has been shown.

This simple concept of noise is probably the least appreciated of all the important concepts. Yet noise is often the thing which makes or breaks an application. Misunderstandings over noise account for a large percentage of all long distance telephone calls for help — particularly when the value of β is not appreciated.

The typical schematic representations of noise voltage and current are shown in Fig. 4. Any noise current present will inherently be balanced out by an equal and opposite current forced through the feedback impedance, Z_f, by the amplifier. To re-emphasize: *any* current (from any source whatsoever) which is pumped into the summing junction is effectively transduced by Z_f into an output voltage. And noise current is no exception. On the other hand, any equivalent noise voltage will inherently be bucked out by an equal and opposite voltage swing at the summing point. The amplifier's output must take whatever excursion is necessary to accomplish these two things. Fig. 4B may make this cause-and-effect easier to visualize. Consider the amplifier's input terminal to be always essentially at ground potential. To accomplish this, the amplifier's output must somehow move the summing point an equal and opposite voltage swing. But the attenuation, β, lies always between the output and the summing point. Thus

the amount the output must move is

$$\Delta e_o \cong e_{NOISE} \times \frac{1}{\beta} \tag{7}$$

where Δe_o = that component of output resulting from the equivalent noise voltage in series with the summing point.

e noise = equivalent noise voltage in series with summing point (see Fig. 4.)

2. "flicker," or random fluctuations in the low frequency range.
3. "power line noise," spurious equivalent noise voltages at power line frequency and its harmonics.
4. "high-frequency noise," all spurious voltages unrelated to and above the power line frequency.

Noise *currents;* i. e., spurious unwanted currents,

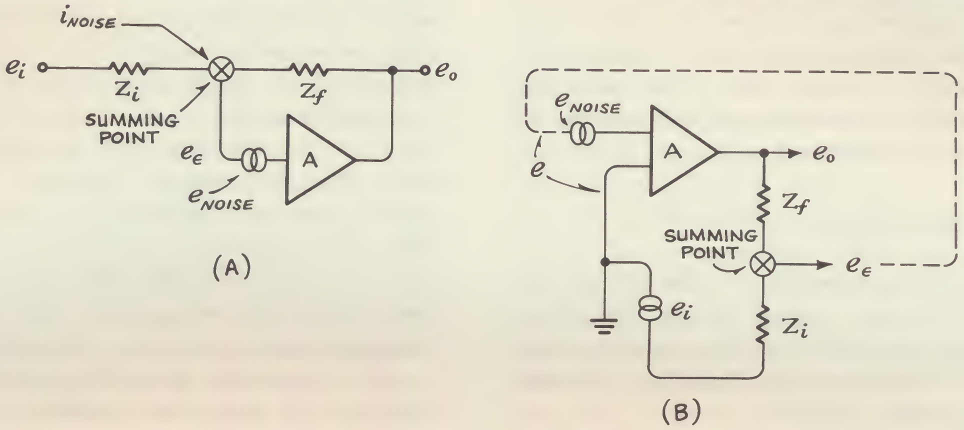


FIGURE 4. EQUIVALENT NOISE CIRCUIT CONCEPTS

Drift, or so-called dc offset changes at the summing point, is purely and simply a form of noise in every respect. There is nothing special or mysterious about it except that its frequency is thought of as very much lower as compared to power line or "flicker"* noise — perhaps even thought of as "dc" when the cyclic period runs hours, days, or months. Drift naturally can manifest itself as a "dc" current (change in the leakage resistance of any dc blocking capacitor connected to the summing point, for example). Unfortunately, most purchasers think of "drift" simply as a form of a voltage noise. Typical operational amplifiers exhibit drift current; i.e., an uncertainty "dc" current of 10^{-8} to 10^{-10} amperes, but the high reliability type discussed previously will show about a thousand times less current.

Drift voltages, referred to the summing point, can be gotten down into the 10 to 20 microvolt area for the best class of computor amplifiers. In what might be called the average class, drifts of 50 to 100 microvolts are conventional; in the economy class, one must put up with 100 to 1000 microvolts.

To insure that each of you meet with heart-warming success on your FIRST APPLICATION, give thoughtful consideration to the effect of the following types of noise *voltage:*

1. "dc drift," long term and short term.

* Flicker is a convenient expression of technical slang, usually applied to more or less random pulsation type of noise, typically in the frequency range of 0.1 to 1 cps.

into the summing junction can come from many obvious sources. But the two subtle sources which so often ambush and needlessly destroy the success of an application are:

1. dc (or ac) insulation leakage currents.
2. spurious capacitive coupling between summing point and other circuits, especially ac circuits operating from power lines.

An insulation leakage resistance from the summing point to ground (or shield braid of a cable) does not introduce a spurious input current since it does not contain an active voltage source. However, it does affect the value of β. Such leakage as low as 1 megohm is usually of no significance. However, a leakage resistance of 50,000 megohms (a pretty high value) to a point only 500 millivolts above ground could cause over 100% increase in the output error drift of a good integrator circuit.

In the section just passed covering the treatment of noise, the fundamental circuit philosophy underlying the use of the so-called "positive input" terminal was represented but in a different guise, scarcely recognizable. Note that in Figs. 1B, 3B, and 4B the amplifier is intentionally shown as having a differential input so as to develop the more general case. The positive ("reference") terminal was always shown connected to ground. However, it *could* be connected to any arbitrarily chosen voltage. Choosing ground is simply the special case of

zero emf, usually chosen because the impedance is very low and the voltage is reliably accurate (zero!). Any voltage swing, e_R, on this positive (or "reference") terminal will force the amplifier, in turn, to force the summing point to move in 1 to 1 correspondence with it (it has just been established that the [differential] error voltage, e_ϵ is really insignificant). If this is so, then theoretically, the amplifier could not distinguish between a given voltage applied in series between ground and reference terminal and applied between the negative input terminal and the summing point. This latter case has been developed for equivalent noise voltages, and the relationships can be re-expressed as:

$$\Delta e_o \cong e_R \times \frac{1}{\beta} \tag{8}$$

where e_R = voltage appearing on the positive (or "reference") input terminal and gain is high enough and common mode rejection ratio is high enough to neglect their contribution to error.

The effect of a ground loop voltage, dc or ac, is now seen to be just another source of voltage noise.

When the positive input terminal can be kept at a high impedance level, such as an open grid for example, all sorts of new analog instrument circuits — previously impossible — become practical and economical.

The "Manual for Computing Amplifiers"[1] covers a score of such rarely published applications.

At this point, a word should be said about differential inputs versus single ended inputs.

Electronic operational amplifiers featuring differential input can be divided roughly into two different classes, and quite different classes at that. In the one class can be gathered the simpler, highest reliability, forms of operational amplifiers which are very economical. At least one manufacturer sells such amplifiers as low as $22. Although they are not suitable for the tough, dramatic job previously discussed, nevertheless they perform admirably in many of the typical computor circuits, since they are dc amplifiers with a gain of 15,000 or better and may have very wide band widths. The other general category of operational amplifiers featuring differential inputs is that typical of instrument-type amplifiers in which the drifts and noise are very low indeed, but the cost is high indeed ($500 to $1,000, or higher), and the frequency response typically very narrow. Their reliability is certainly adequate for driving indicating and recording equipments, but, because of their complexity and number of components required in their construction, they are hardly in a class with the former category — let alone their hydraulic or pneumatic counterparts. Most of the non-differential or single-ended types of operational amplifiers are chopper-stabilized. These typically feature fantastic dc gains and drifts ranging anywhere from 1 microvolt per day to as much as a millivolt per day. There seems to be no sharp dividing line between the type which one could class as high-reliability and those which could be classed as typical measurement instrument reliability. Reliability generally goes "hand in glove" with simplicity, and some of such operational amplifiers selling under $100 could be considered approaching the so-called high-reliability level. At the other end of the scale, the highest reliability amplifier and also the highest performance operational amplifier known to the author sells for about $170. It is interesting at this point to note that the typical pneumatic amplifier counterpart sells for about $160. The model the author has in mind is certainly also of the high-reliability class, but the frequency response and overall accuracy are only a very small fraction of that practical with the electronic counterpart.

All the concepts generally necessary for successful application of operational amplifiers are given above. For those interested in digging deeper into the limitations and circuit performance, appendix A is included. It consists of notes on operational amplifiers from a lecture given some time ago before the IRE. In addition to this, Philbrick Researches will be most happy to send you a number of additional reprints on operational amplifiers and computing art, in general. Other manufacturers offer similar literature.

Again, we have concentrated our explanation on the electronic model. But, the considerations and the philosophies are the same for hydraulic amplifiers, or pneumatic. Their performance may not be so dramatic, nor their characteristics so cleancut, but their reliability can be quite as good. In fact, it would be presumptuous to imply that the pneumatic controller is not basically more reliable in many applications. Certainly, it is a far simpler device.

IV OPERATIONAL AMPLIFIERS AT WORK

Analog Methods is simply another name for *"the activation of passive circuits* with operational amplifiers." Most operations become almost self-evident when the amplifier itself is understood and the local ground rules for its application are appreciated. Calculation of the exact performance can then follow from the considerations presented in Section II. When visualizing the operation of any of the circuits to follow, always start with the premise:

> If the amplifier output is *anywhere* within its rated range, then the summing point error, e_ϵ, in Figs. 1, 3, and 4 must be negligible.

This "backwards" type of thinking is a necessary discipline when one is putting the operational amplifier to work.

The discussion centers about various electrical amplifiers of the operational type. Actual circuit connections vary depending on whether the actual operational amplifier is a magnetic amplifier, or a conventional tube or transistor type (basis of the following descriptions), or a carrier modulated (chopper) type with full differential floating input, again either tube or transistor type. Each type has special advantages to offer, each can perform certain tricks which make it an obviously better choice in one application or another.

Reliability is a primary concern to the process industry, often more important even than price. Short cuts to it are rarely achieved. Real, honest reliability is generally expensive. It is the author's experience in control system design that failures are most often due to mechanical, workmanship, or quality control considerations, and least of all to whether transistors, magnetic amplifiers, or vacuum tubes have been used (when competent engineering effort has placed emphasis on the best design practice).

Inversion

Inversion, or reversal of signals, is one of the most common operations performed in the electronic analog art. It is also about the least demanding, since it amounts simply to a 1:1 amplification, that is, $Z_i = Z_f$, referring to Fig. 1.

Addition & Subtraction

The circuit philosophies involved in summation were developed in detail in Section II. Operational amplifiers can take algebraic *sums;* they cannot subtract. To subtract one signal from another, one must use an inverter to change the sign, or sense, of the subtrahend and then add the inverted subtrahend to the minuend. Unfortunately, this costs one additional operational amplifier. However, to subtract one pressure from another in a pneumatic device, the "negative sign" bellows can simply be placed on the opposite side of the force-balance arm or plate, note Figs. 5 and 6.

Multiplication

There are four basic types of multipliers in common use in the analog field (although others exist):

1.) Pulse width-pulse height modulation circuit, the so-called "height-width" type.
2.) Philbrick matrix multiplier circuit.
3.) "Quarter-Square" type.
4.) Servo-multipliers.

Servo multipliers can be very accurate indeed when their potentiometers and servo systems are in good order. However, they are usually considered to be more easily deranged than the other types mentioned. Their interest to the process industries is likewise thought to be less than that of the height-width and Philbrick matrix type circuits.

The "quarter-square" type multipliers have been in use for some time, and are advantageous at frequencies somewhat higher than is practically obtainable with height-width or Philbrick matrix type multipliers. The scheme of operation is not elaborate but the circuit complexity inherent in the devices approaches that of the "height-width" type. These two types are necessarily more complicated than the Philbrick matrix. To accomplish the multiplication, the "quarter-square" device, in one part of the circuit, adds the two voltages (representing the two variables). Their sum is then squared (by any of several methods of function generation). In another part of the circuit, one of the two inputs is subtracted from the other input and this difference is squared. The square of the difference is then subtracted from the square of the sum. The remainder is four times the product of the two variables, hence the "quarter-square" terminology:

$$\frac{(x+y)^2-(x-y)^2}{4} = xy$$

Several methods of squaring exist, but the most popular one appears to be the diode-resistor nonlinear network method (to be covered later). Although its accuracy leaves something to be desired, it at least can be made to give usable performance out to frequencies of the order of 10 kc to 100 kc.

The height-width multiplier and the Philbrick matrix multiplier are both inherently capable of very high accuracy in the best quadrant. As an arbitrary opinion, perhaps 0.01% of full output is a reasonable limit of practicality at present for both schemes. Adjusted for the best two quadrants, the accuracy possible is still almost as good. However, taken in all four quadrants, the worst case error is likely to be over four times this value. In terms of practical, highly reliable equipment available on the market both schemes cost well over a thousand dollars (when all the necessary accessories are included), and both are capable of better than 0.1% accuracy.

Fully transistorized multipliers utilizing the Philbrick matrix philosophy have recently come on the market at a cost well under $1000. Typical accuracies overall are reported to be well under ½% This type of multiplier is of unusual interest in the process control field because it actually handles multiplication and division simultaneously:

$$\text{output} = \frac{xy}{z}$$

where: x, y, and z are three independent variables. When one of the three is not needed, a fixed reference voltage is switched into its position. For this reason the Philbrick matrix type is usually referred to as a "Multiplier-Divider." Division can be accomplished with other multipliers by using the entire multiplier as part of the feedback impedance, Z_f, around another operational amplifier as shown in Fig. 9.

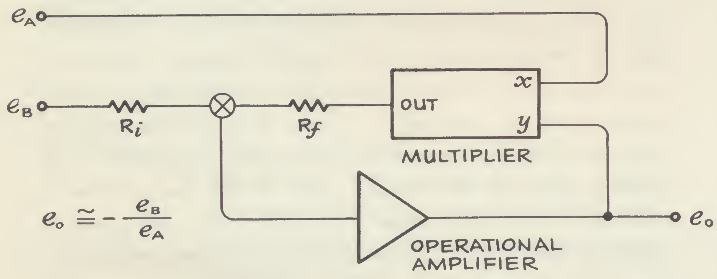


FIGURE 9. DIVIDING CIRCUIT

Integration

Integration can be performed with electronic operational amplifiers to dazzling accuracy because of two inherent features: 1) The remarkable degree of isolation between feedback and input circuit elements, and 2) Signals come in as voltages, are transduced to currents by the input impedance, Z_i, and these currents are then transduced back into an output voltage by the feedback impedance, Z_f. The voltage across an ideal capacitor is the true integral of the current flowing into or out of it divided by the capacitance. Therefore it constitutes an ideal transducer, or feedback impedance, and the output voltage is the true integral of the *sum* of the input currents divided by the capacitance. (See Fig. 3):

$$e_o = \frac{1}{C_f}\int i_i \, dt$$

where i_i = input current in amperes
C_f = feedback capacitor in farads

Or for the simplest circuit (See Fig. 1)

$$e_o = \frac{1}{T}\int e_i \, dt$$
$$\textit{where } T = R_i C_f$$

The practical limits on the length of time over which a dependable integration can be performed (or the solution held in memory) is limited by the spurious dc currents previously discussed under "Operational Amplifiers." If the high reliability amplifier used as an example were operated as a 10 second integrator (R_i = 1 megohm and C_f = 10μf), the system designer should probably allow for 2 x 10^{-11} amperes as unwanted, spurious current (dc noise) resulting from 20 microvolts drift (across the megohm input resistor). Suppose a 1% error in the output were tolerable. This would be 1% of 1000 volts, or 1 volt. Therefore one would have to limit the integrating time to 5 x 10^5 seconds, or about 6 days. This may seem adequate for most control systems, *but* it represents the practical upper limit for the very best of present day equipment. Furthermore, it requires a capacitor whose internal leakage time constant is better than a year and a half. Although such capacitors have recently been offered commercially, their cost would likely exceed that of the operational amplifier itself. The above is cited as representing an extreme case. However, capacitors have been routinely and economically available for several years which are about one tenth this good. Those would permit a 14-hour integration with less than 1% error on the same basis.

True integrators must be "clamped," or somehow reset to the initial conditions value after the problem solution has been read out, and before a new solution can be started. Relays are currently popular among analog computor manufacturers for this application. Certain types are considered quite reliable. For example, the mercury-wetted contact types made under Western Electric Co. License are, in general terms, one of the most reliable electrical components normally found in control systems. Life tests, now in progress at C. P. Clare Co., are running 60 operations per second switching 5 amperes. This is a staggering total of 5,-184,000 operations per day for over 4 years (over 8 billion)!!!

Clamping to zero volts (Fig. 10A) is not only the most reliable circuit, it is also far less complex a circuit and very much faster. Initial conditions can usually be added in on the next stage following. A ten millisecond total time for clamping and discharging to zero volts is quite practical.

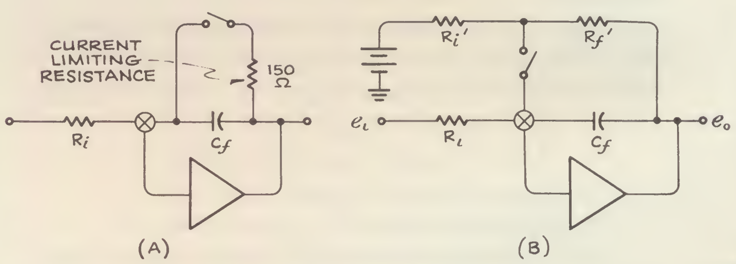


FIGURE 10. CLAMP CIRCUITS

However, clamping to a definite voltage can be done reliably and accurately also. The practical penalty paid is simply clamp and reset time. In this case the amplifier must furnish all the current. As can be seen in Fig. 10B, this puts a minimum allowance on the resistor, R_f, and thus also puts a limit on the speed with which C_f can be reset. If $R_f C_f = T_f$, the reset time constant, it will require about 5 times this length of time to reset to 1%. Thus, if R_f is 20,000 ohms, a typical choice on the low side, and C_f is the 10μf previously discussed, it will take about one second to reset to 1% or about 1½ seconds for 0.1%. By contrast, if it is clamped to zero as shown in Fig. 10A it will take about 8 milliseconds for 1% or about 12 milliseconds for 0.1%.

An important item on the system designer's check list is the freedom from leakage directly across the integrating capacitor. In the arrangement shown in Fig. 10A and 10B, the circuits are elementary and the freedom from leakage resistance around the capacitor is limited to that of the relay. (Generally speaking, relays do not approach the freedom from leakage of the best capacitors, (ten million megohms and higher), and certainly not the relays of the higher reliability mercury-wetted types.) But this defect can be readily overcome by the use of the Philbrick dc guard circuit, or variations of it. The mechanics of this current are not so important now as the fact that the leakage must be considered in the system design. For those cases where it is necessary to defeat this leakage, circuit techniques exist which can deal satisfactorily with leakage as low as 10 megohms!

Pneumatic control systems can make use of long lags to approximate a short term integration. The restriction and airtank of Fig. 7 is a direct analogy to the simple lag circuit of Fig. 11A, and its more practical analog circuit equivalent, Fig. 11B. This satisfies most control stabilization needs with simple pneumatic circuitry. Lags of hours, or even days, do not pose a real difficulty. Furthermore, clamping is not an inherent necessity in a "lag" operation. But neither is it usable as a true integrator, capable of accurate, long term memory.

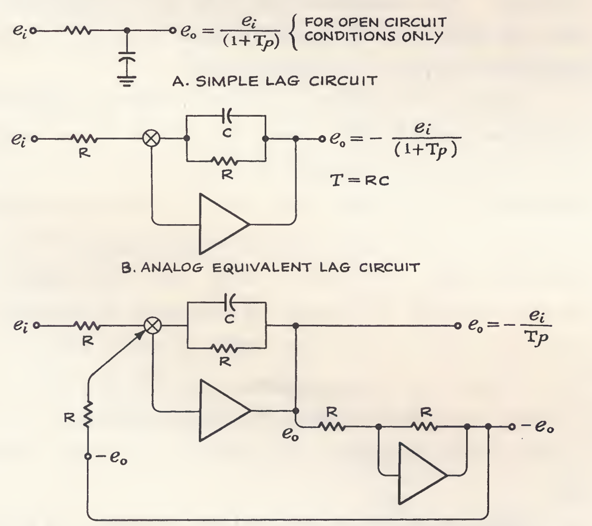
It can be compensated by an auxiliary pneumatic circuit, shown in analogy in Fig. 11C. The auxiliary circuit simply computes what the current in the shunting resistor must be and supplies it back into the summing point as positive feedback.

If the compensation of this analogous loss current were perfect, the circuit would yield a theoretically true integration. But if the total error of compensation were reliably less than 0.1% under any and all conditions (not an impractical accuracy) it could still perform an integration over a period ten time constants long, ie., 10 x RC, and be within 1% of the true integral. This performance may not be very impressive compared to its electronic counterpart, but it could do certain jobs, and do them reliably.

Before leaving the subject of integration, mention should be made of the fact that a true double integration can be performed electronically with only one operational amplifier. This circuit is shown in Fig. 12.

Differentation

Differentation may be essentially the inverse of integration, but in practical circuitry new problems are presented, and some of the old ones lessened. First of all, the signal itself usually contains noise components at frequencies much higher than the frequencies of interest. In those rare applications where it does not, the control or computing devices handling the signal may



C. ANALOG LAG CIRCUIT COMPENSATED TO OPERATE AS AN INTEGRATOR

FIGURE 11. ANALOG OF AIR TANK & RESTRICTION

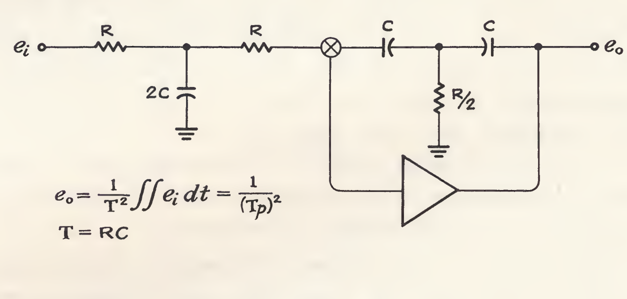


FIGURE 12. DOUBLE INTEGRATOR CIRCUIT

add such noise. The actual absolute value of such noise may be truly negligible, whereas the true derivatives may have quite objectionable magnitudes. A so-called "differentiator" which somehow does not have "noise troubles" is an equipment with degraded frequency response. The only approach recommended is to degrade it intentionally by just the right amount, no more, no less.

"derivative action" is shown in Fig. 7. This serves the purpose satisfactorily in the applications for which it was designed, and it does it reliably. However, such circuits do not yield the true derivative. They are analogous to installing a resistor in Fig. 13C in the feedback path (shown in dotted lines) around the forward amplifier, whose value is equal to or less than the input resistor.

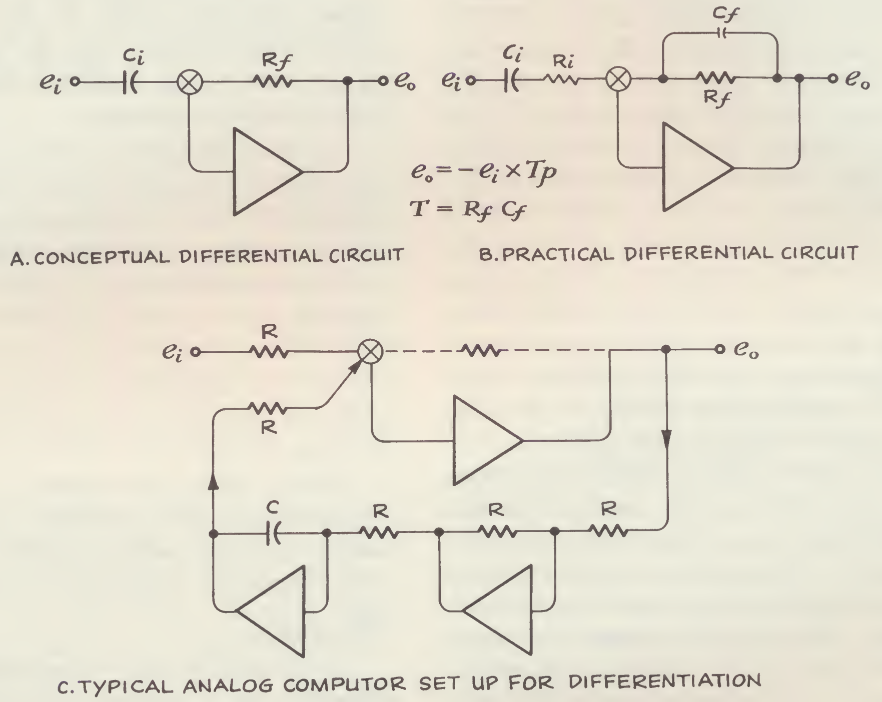


FIGURE 13. DIFFERENTIATOR CIRCUIT

Fig. 13A shows the conceptual schematic of a differentiator. Fig. 13B shows a practical circuit whose mechanics are almost self-evident. When

$$R_i C_i = R_f C_f = T_{co}$$

the circuit provides the maximum accuracy up to the cut-off frequency set by the choice of the cut-off time-constant, T_{co}. Also, it provides the lowest noise content (for 12 db/octave filter action). The extremes of performance of which it is capable are fully the equal of the integrator previously discussed.

In large analog computor installations when ultimate performance is not a requirement it is often more convenient to differentiate by connecting an inverter plus integrator in the feedback path around a third amplifier as shown in Fig. 13C. However, this always results in some *increase* in noise for a given bandwidth as compared with the single amplifier arrangement recommended in Fig. 13B. For some reason, there is a widespread misconception about this.

The conventional pneumatic method of providing

Function Generation

Function generation in most pneumatic control systems is accomplished by specially cut cams or mechanical linkages. Its discussion does not seem appropriate to the real intent of this paper because no special concepts are involved, least of all pneumatic.

There are several practical ways of generating functions electrically. However, there is one method that stands out in its possible applications to the process industries. This method consists of activating nonlinear networks is endless, but the basic philosophy is about the same for all. Referring to Fig. 14A, the amplifier is used to position the output as necessary to maintain the summing point at zero (ground) potential while the input voltage, e_i, is manipulated. In this way, the output voltage, e_o, can be forced by the amplifier to provide a very wide latitude of functions of the input voltage, e_i.

Silicon diodes are the nonlinear elements of primary interest. They are extremely sturdy, reliable, and nearly

foolproof when applied within their ratings. Very little selection even is needed to get groups of certain diodes to track within 20 millivolts over a 3 to 1 change in current taken in combination with a ±25°C change in ambient temperature. When compared with full scale voltages of 100 volts, a conceivable comparison accuracy

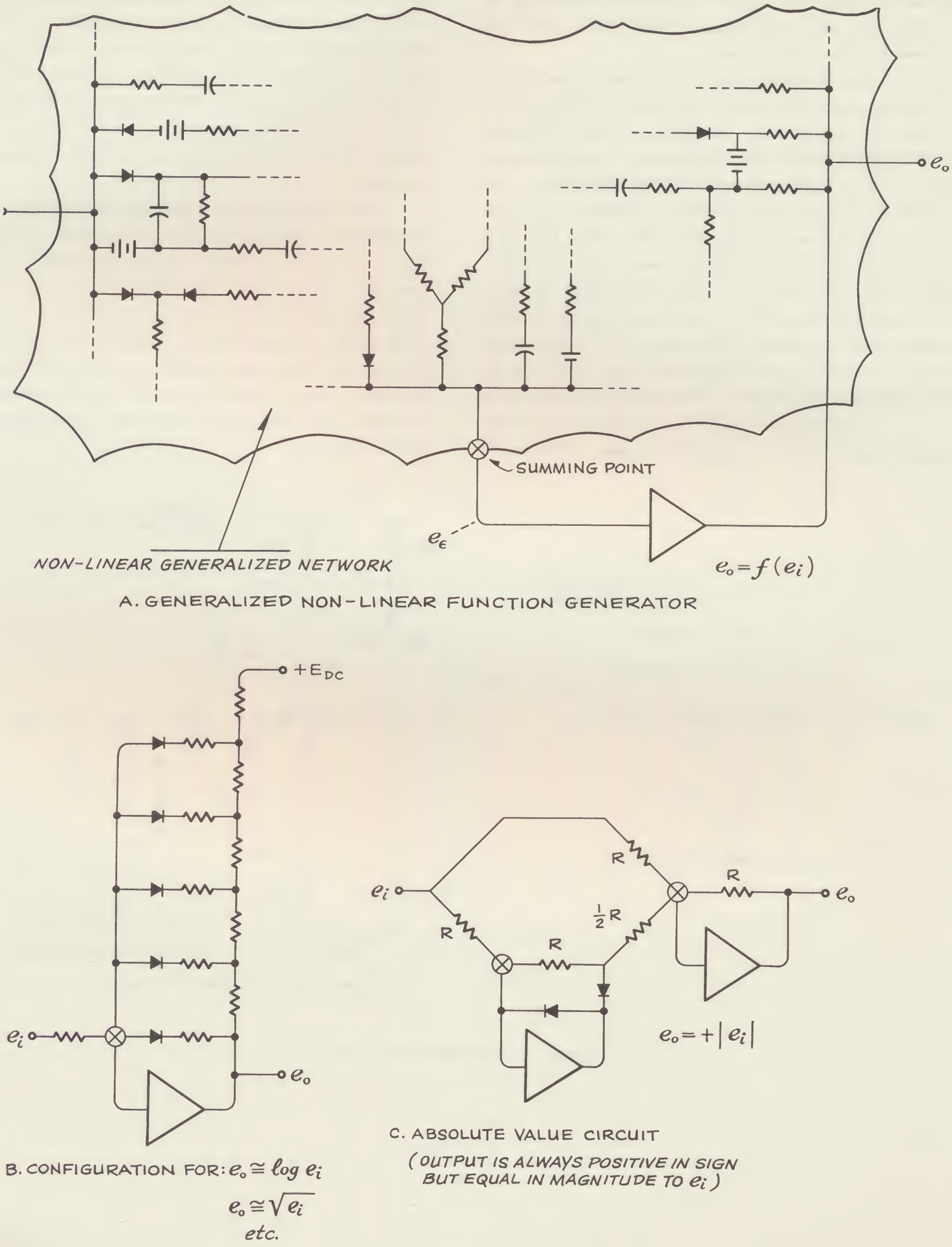


FIGURE 14. NON-LINEAR FUNCTIONS

or repeatability of substantially better than 0.1% is a clear, inherent possibility. When the entire diode drop must be accepted as an error (cannot practically be compensated out), many types permit this drop to be kept well below 400 millivolts, or 0.4% of a 100 volt range even for this severe case.

Some of the imperfections of the diodes can be removed almost completely by activating the diodes themselves with an operational amplifier. The "absolute value" (idealized full-wave rectification) circuit shown in Fig. 14C is a good example of this technique. To appreciate the degree of perfection possible, one need only note that the summing points can never be more than a few microvolts dc different from zero (ground) potential while the amplifier output is within working limits. Therefore, one diode must at all times be fully conducting and the other fully turned off or blocking. When the current *into* the summing point just exactly switches polarity, the output of the amplifier proper will *jump* to the opposite polarity, to the magnitude necessary to establish conduction in the other diode. When the diode connected to the junction of R and ½R is blocking, its reverse current can quite reliably be kept below 10^{-8} amperes. Since the other ends of both of these resistors are connected to summing points, the voltage across these resistors must be negligible. If R = 50K, then the error *at the output* must be only 333 microvolts due to the diode leakage, or about 3 parts per million error out of the full range of 100 volts. Once again, the dc error is seen to be limited in practical circuits to the tolerance one can afford to buy in their resistors.

To further stimulate thinking in nonlinear function possibilities, consider the "bound" circuits of Fig. 15. In both of these, the diodes are used passively, hence their forward voltage drops contribute some error. However, in the symmetrical bound circuit shown in Fig. 15B the bulk of this voltage drop is compensated out at the take-over point (near zero potential) and at the bound point, the bulk of the forward drop can be calibrated out. Actually, this circuit can be made even more accurate than ½%.

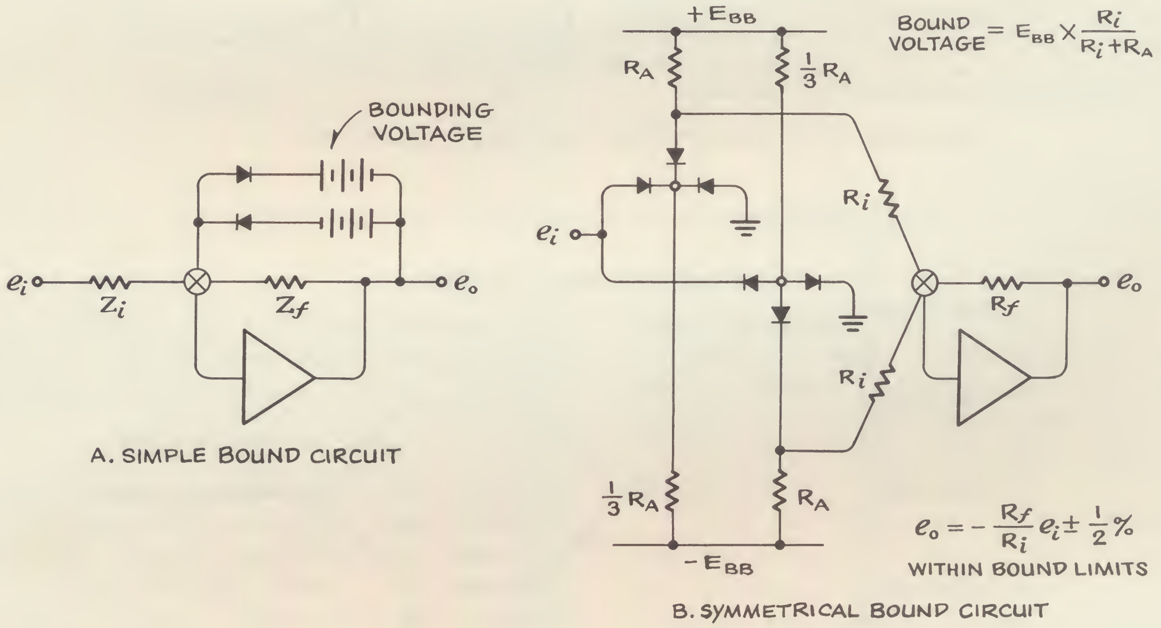


FIGURE 15. BOUNDING (LIMIT) CIRCUITS

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